# Barem de corectare - clasa a III-a

1. a) $806 : 2 + 574 \cdot 9 = 459$ — 1p.
$300 : 6 + 790 : 10 = 129$ — 1p.
$(459 - 129) \times 3 = 990$ — 1p.
$990 : 5 = 198$ — 1p.

b) $n \times n : n = n$ — 1p.
$n \times 0 = 0$
$n \times 1 = n$ } -1 p
$n : n = 1$
$n + 0 - n + 1 = 1$ — 1p.

2. - Determinarea literelor
$M = 5$; $A = 1$; $T = 4$, $E = 6$ } 7 p
$I = 3$, $C = 2$
Ordinea $T = 4$...

3a) $x + x + 1 + y + y + 1 = 748 \Leftrightarrow$ } -1p.
$x + y = 373$
$x + 1 - y = 100 \Leftrightarrow x - y = 99$ ... - 1p.

$x + y = 373$
$x - y = 99$ $\Rightarrow 2x = 472 \Rightarrow x = 236$ } - 1 p

$\Rightarrow y = 137$
$\Rightarrow 236, 237, 137, 138$ — 1p.

b) Presupunem că cele 28 monede sunt de 3 lei $\Rightarrow 28 \times 3 = 84$ lei
$100 - 84 = 16$ lei $16 : 2 = 8$
Deci 8 monede de 3 lei dein de 5 lei
și 20 monede rămân de 3 lei
Verificare $8 \times 5 = 40$. } 100 lei — 2 p.
$20 \times 3 = 60$ }

Analog $29 \times 3 = 87$ lei
$100 - 87 = 13$ lei — 1 p.
Răspuns ne

Barem de corectare - clasa a IV-a

1.a). Adunăm relațiile și obținem

$2013a + 2013b + 2013c = 12078 \Rightarrow$ | 1p.

$a + b + c = 6$

$2011a + b + c = 2016$ | $\Rightarrow 2010a = 2010 \Rightarrow a = 1$ | 1p

$a + b + c = 6$

$a + 2011b + c = 4026$ | $\Rightarrow 2010b = 4020$ | 1p.

$a + b + c = 6$ $\quad b = 2$

$c = 3$ - 1p.

b) $A+7 + A+8 + A+9 + A+10 + A+11 = 495$ | - 1p.

$5A + 45 = 495 \Rightarrow 5A = 450 \Rightarrow A = 90$ | - 1p.

$\Rightarrow 97, 98, 99, 100, 101$ - 1p.

$97 + 98 + 99 + \dots + 115 = 2014$ - 1p.

2a) $1000a + 100b + b + 100a + b + 10b + a + 2b + a = 2014$ - 2p

$\Rightarrow 1102a + 114b = 2014 \Rightarrow 551a + 57b = 1007$ - 1p

- 1p

$a = 1 \Rightarrow b = 8.$

b) $8x + 1 + x = 235$ - 1p.

$9x = 234$ | - 1p.

$x = 26$

Fratică s-a clasat pe locul 27 - 1p.

3a) $a + b = 1023$ - 1p

$c = 97$ - 1p.

$3a + 3b - 7c = 3 \cdot (a+b) - 7c =$

$= 3 \cdot 1023 - 7 \cdot 97 = 2390.$ - 1p.

b) $\overline{abcde} : 2014 = x \Rightarrow \overline{abcde} = 2014 \cdot x + 13.$ - 1p.

$\vdots$

$13$

$\overline{abcde} = 2014 \cdot 4 + 13$ F

$= 2014 \cdot 5 + 13 = 10070 + 13 = 10083$ (A) | - 2p.

$\dots$

$= 2014 \cdot 49 + 13 = 98699$ (A)

$= 2014 \cdot 50 + 13 = 100713$ F - 1p.

Există $49 - 4 = 45$ (numere)

# Barem de corectare - clasa a V^a.

1a) $4x < 7x$ | $4x = 4x$
$7y = 7y$ | $4y < 7y$ | 3 p.
$2013 < 7(x+y) \Leftrightarrow$ | $4(x+y) < 2013 \Leftrightarrow$
$x+y > \frac{2013}{7} > 287$ | $x+y < \frac{2013}{4} < 504$.

$\Downarrow$ — 1 p.

$287 < x+y < 504$.

b) $343^{671} = (7^3)^{671} = 7^{2013} = 7^3 \cdot 7^{2010} = 343 \cdot 7^{2010}$ — 1p
$16^{1007} = (4^2)^{1007} = 4^{2014} = 4^4 \cdot 4^{2010} = 256 \cdot 4^{2010}$ — 1p.
$\Leftrightarrow 343^{671} > 16^{1007}$ — 1p.

2a) ~~Folosim~~ metoda reducerii la absurd
$\Rightarrow a < 672$, $b < 672$, $c < 672$ $\Leftrightarrow$ $a \leq 671$, $b \leq 671$, $c \leq 671$ $\Rightarrow a+b+c \leq 2013$ (F) | 4p.
$\Rightarrow$ cel puțin un număr este mai mare ca 671

b) Presupunem că fracția se simplifică prin d
$\Rightarrow d/38n+5 \Rightarrow d/53(38n+5) \Rightarrow d/2014n+265$ | 3p.
$d/53n+7 \quad d/38(53n+7) \quad d/2014n+266$
$\Rightarrow d/1 \Rightarrow$ fracția ireductibilă

3a) din ii) $\Rightarrow a+2c = d-b$
iii) $\Rightarrow 10a-10c = 6+d-b \Leftrightarrow 10a-10c = 6+a+2c \Rightarrow$
$3a-4c = 2 \Rightarrow a \in \{2,4,6,8\}$
Fie $a=2 \Rightarrow c=1 \Rightarrow d-b = a+2c = 4 = 4-0 \Rightarrow 2014$
$a=4 \Rightarrow 4c = 10$ F. $= 5-1$ (F) $- c=b$
$a=6 \Rightarrow c=4 \Rightarrow d-b = 14$ (F) $= 6-2$ (F) $- a=b$
$a=8 \Rightarrow 4c = 22$ F $= 7-3 \Rightarrow 2317$. 4p.
$= 8-4 \Rightarrow 2418$.
$= 9-5 \Rightarrow 2519$.
$S = 2014+2317+2418+2519 = 9268 = 331 \cdot 28$

b) i) $\frac{1}{c+1}\left(\frac{ac+2}{1} + \frac{a-2}{ac+2}\right) = \frac{1}{c+1} \cdot \frac{ac+2+a-2}{ac+2} = \frac{a(c+1)}{(c+1)\cdot(ac+2)} = \frac{a}{ac+2}$ | 3p.

ii) $\frac{2014}{6041} = \frac{1}{3}\left(1 + \frac{1}{6041}\right) = \frac{1}{3} + \frac{1}{18123}$

$6041 \quad 2014 = 2$
$4028$
$2013$

# Barem de corectare - clasa a VI-a

1.a) Scădem 45 format din $1+2+3+\dots+9 \Rightarrow$

$\left(\frac{x+3}{2}-1\right)+\left(\frac{x+7}{3}-2\right)+\left(\frac{x+13}{4}-3\right)+\dots+\left(\frac{x+91}{10}-9\right)=0 \Leftrightarrow$

$\frac{x+1}{2}+\frac{x+1}{3}+\dots+\frac{x+1}{10}=0 \Leftrightarrow$

$(x+1)\left(\frac{1}{2}+\frac{1}{3}+\dots+\frac{1}{10}\right)=0 \Rightarrow x+1=0 \Leftrightarrow x=-1$ (4p.)

b) i) Fie $(3a+11b):9 \Rightarrow b:3 \Rightarrow \boxed{b=3p} \Rightarrow 3(a+11p):9 \Rightarrow$

$(a+11p):3 \Rightarrow a+11p=3h \Rightarrow a=3h-11p.$

Atunci $6a-2b=6a-6p=6(a-p)=6(3h-12p)=$

$=18(h-4p):18 \Rightarrow P:9\cdot 18=162.$ (2p.)

ii) Fie $(6a-2b):9 \Rightarrow b:3 \Rightarrow b=3K \Rightarrow 6a-6K=$

$=6(a-K):9 \Rightarrow a-K=3Q \Rightarrow a=3Q+K$

Atunci $3a+11b=3(3Q+K)+33K=(9Q+36K):9$

Cum $(6a-2b):2 \Rightarrow P:2\cdot 99=162.$ (1p.)

2a) Împărțim inegal ~~toba~~ prin $abcd \Rightarrow$

$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{1767}{abcd} \le 10$

Suma maximă dacă $a,b,c,d$ minime $\Rightarrow a=2, b=3, c=5, d=7$ (4p.)

$\Rightarrow S=9\frac{62}{105}<10$ (A).

b) i - clasa a V-a

ii - $\frac{2014}{8055}=\frac{1}{4}\left(1+\frac{1}{8055}\right)=\frac{1}{4}+\frac{1}{32220}.$ (3p.)

3) Se poate folosi proprietatea punctelor de pe bisectoarea unui unghi sau congruențe de triunghiuri.

$\triangle BAD \equiv \triangle BFD$ (I.U) $\Rightarrow (AB) \equiv (FB)$

$\triangle AIB \equiv \triangle FIB$ (L.U.L) $\Rightarrow (AI) \equiv (FI)$

$\triangle AID \equiv \triangle FID$ (L.L.L) $\Rightarrow m(\widehat{DAI}) = m(\widehat{DFI}) = 45^\circ \Rightarrow m(\widehat{IFG}) = 45^\circ$ (3p.)

Analog

$\triangle CAE \equiv \triangle CGE$ (I.U) $\Rightarrow (AC) \equiv (GC)$

$\triangle AIC \equiv \triangle GIC$ (L.U.L) $\Rightarrow (AI) \equiv (GI)$

$\triangle AIE \equiv \triangle GIE$ (L.L.L) $\Rightarrow m(\widehat{IAE}) = m(\widehat{IGE}) = 45^\circ \Rightarrow m(\widehat{IGF}) = 45^\circ$ (3p.)

$\triangle IGF \Rightarrow m(\widehat{FIG}) = 90^\circ.$ (1p.)

# Barem de corectare - clasa a VII a

1) a) Dacă $p=2 \Rightarrow K^2=7^2$ (A)

Dar $24p+1$ este impar $\Rightarrow K=2t+1$ și ecuația devine $24p+1=(2t+1)^2 \Leftrightarrow 24p+1=4t^2+4t+1 \Leftrightarrow$

$6p=t(t+1) \Rightarrow \begin{matrix} 6\cdot 5 \\ 6\cdot 7 \end{matrix}$ sau $\begin{matrix} p=5 \\ p=7 \end{matrix}$

Discuție

- din $6p=t(t+1) \Rightarrow t=6u$ sau $t+1=6u$.

$\Rightarrow 6p=6u(6u+1) \Leftrightarrow p=u(6u+1) \Rightarrow u=1 \Rightarrow p=7$

$\Rightarrow$ din $t+1=6u \Rightarrow t=6u-1 \Rightarrow 6p=6u(6u-1) \Rightarrow p=u(6u-1) \Rightarrow u=1 \Rightarrow p=5$ — 4p.

b) i) $\frac{1}{n+1}\left(1+\frac{n-2}{n\cdot u+2}\right)=\frac{1}{n+1}\cdot\frac{n\cdot u+2+n-2}{n\cdot u+2}=\frac{n(n+1)}{(n+1)(n\cdot u+2)}=\frac{n}{n\cdot u+2}$

ii) $\frac{1151}{2014}=\frac{1}{2}\left(1+\frac{144}{1007}\right)=\frac{1}{2}\left[1+\frac{1}{7}\left(1+\frac{1}{1007}\right)\right]=\frac{1}{2}+\frac{1}{14}+\frac{1}{14098}$ — 3p.

2a) $\Rightarrow a(a+1)=p^{2^k}+2$ din $a(a+1)=2q \Rightarrow p=2 \Leftrightarrow$

$a(a+1)=2^{2^k}+2 \Leftrightarrow a(a+1)-2=2^{2^k} \Leftrightarrow a^2+a-2=2^{2^k} \Leftrightarrow (a+2)(a-1)=2^{2^k}$

Dar $a+2$ și $a-1$ sunt de parități diferite.

Cum $a+2 \geq 2 \Rightarrow a-1 \geq 1 \Rightarrow a=2 \Rightarrow 2^{2^k}=2^2 \Rightarrow k=1$. — 3p

b) folosim ineg. mediilor $\frac{x+y}{2} \geq \sqrt{xy} \Leftrightarrow$

$$\left.\begin{array}{l} \frac{1}{2}\left(\frac{abd}{c}+\frac{abc}{d}\right) \geq \sqrt{\frac{abd}{c}\cdot\frac{abc}{d}}=ab \\ \frac{1}{2}\left(\frac{bcd}{a}+\frac{abc}{d}\right) \geq \sqrt{\frac{bcd}{a}\cdot\frac{abc}{d}}=bc \\ \frac{1}{2}\left(\frac{bcd}{a}+\frac{acd}{b}\right) \geq \sqrt{\frac{bcd}{a}+\frac{acd}{b}}=cd \\ \frac{1}{2}\left(\frac{acd}{b}+\frac{abd}{c}\right) \geq \sqrt{\frac{acd}{b}\cdot\frac{abd}{c}}=ad \end{array}\right\} \overset{+}{\Rightarrow} c\mathcal{L}.$$ — 4p.

3) a) Fie $AC \cap BD=\{O\}$.

$\left.\begin{array}{l} \triangle AOB \overset{T.P.}{\Rightarrow} AB^2=OA^2+OB^2 \\ \triangle DOC \overset{T.P.}{\Rightarrow} DC^2=OC^2+OD^2 \end{array}\right| \overset{+}{\Rightarrow} AB^2+CD^2=OA^2+OB^2+OC^2+OD^2$

$\left.\begin{array}{l} \triangle AOD \overset{T.P.}{\Rightarrow} AD^2=OA^2+OD^2 \\ \triangle BOC \overset{T.P.}{\Rightarrow} BC^2=OB^2+OC^2 \end{array}\right| \overset{+}{\Rightarrow} AD^2+BC^2=OA^2+OB^2+OC^2+OD^2$ $\Rightarrow$ a) — 3p.

b) Construim $CM \perp AB \Rightarrow MB=AB-CD$.

$\triangle BCM \overset{T.P.}{\Rightarrow} BC^2=(AB-CD)^2+AD^2=AB^2+AD^2+CD^2-2AB\cdot CD \Rightarrow$

$BC^2-AB^2=AD^2+CD^2-2AB\cdot CD$ (*).

din a) $\Rightarrow AD^2+BC^2=AB^2+CD^2 \Leftrightarrow BC^2-AB^2=CD^2-AD^2$ $\overset{\text{înl.}}{\Rightarrow}$

$CD^2-AD^2=AD^2+CD^2-2AB\cdot CD \Leftrightarrow AD^2=AB\cdot CD \Leftrightarrow$

$AD=\sqrt{AB\cdot CD}$. — 4p.

Trebuie să dem. $AB+CD>2AD \overset{\text{înl.}}{\Rightarrow}$

$AB+CD>2\cdot\sqrt{AB\cdot CD} \Leftrightarrow \left(\sqrt{AB}-\sqrt{CD}\right)^2>0$ (A).

# Barem de corectare – clasa a VIII-a

1.a) $a^2b^2 - 11a^2 - 11b^2 + 121 = 2014 \Leftrightarrow (a^2-11)(b^2-11) = 38\cdot 53 \Rightarrow$

$\begin{cases} a^2-11 = 38 \\ b^2-11 = 53 \end{cases} \Leftrightarrow \begin{matrix} a_{1/2} = \pm 7 \\ b_{1/2} = \pm 8 \end{matrix} \Rightarrow S = \{(7,8);(-7,-8);(7,-8);(-7,8)\}$

$\begin{cases} a^2-11 = 53 \\ b^2-11 = 38 \end{cases} \Rightarrow \begin{matrix} a_{1/2} = \pm 8 \\ b_{1/2} = \pm 7 \end{matrix} \Rightarrow \dots$

$a^2 - 11 = 19$

$b^2 - 11 = 106$ (F)

4 p.

b) Fie $f(x) = ax + b$, $a, b \in \mathbb{Z}$. $\Rightarrow \begin{matrix} f(x-2) = a(x-2)+b \\ f(x+2) = a(x+2)+b \end{matrix}$ | cube $\Rightarrow$

$[a(x-2)+b]\cdot[a(x+2)+b] = x^2 - 2x - 3 \Leftrightarrow a^2x^2 + 2abx + b^2 - 4a^2 = x^2 - 2x - 3.$

$\dots (a^2-1)x^2 + (2ab+2)x + b^2 - 4a^2 + 3 = 0 \quad (\forall) x \in \mathbb{R}$

Dacă $a^2 - 1 \neq 0$, atunci ec. are max. 2 soluții $\Rightarrow$

$\Leftrightarrow a^2 - 1 = 0 \Leftrightarrow a_{1/2} = \pm 1 \Rightarrow b_{1/2} = \mp 1 \Rightarrow f_1(x) = x+1$

Analog $2ab+2 = 0$ $\qquad f_2(x) = -x+1$

3 p.

$d = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ C

2a) Deoarece $2x+1 = n^2$ și faptul că printre numerele $2x+2, 2x+3, \dots, 3x+2$ (F) $k^2 \Rightarrow 3x+2 < (n+1)^2$

$2x+1 = n^2 \Rightarrow x = \frac{n^2-1}{2}$ cube $\Rightarrow 3\cdot\frac{n^2-1}{2} + 2 < (n+1)^2 \Leftrightarrow$

$\dots \Leftrightarrow n(n-4) < 1 \Rightarrow n > 4$ nu convine

$\Rightarrow n \in \{1,2,3,4\} \Rightarrow n=1 \Rightarrow x = 0$ (F)

$n=2 \Rightarrow 2x+1 = 4$ (F)

$n=3 \Rightarrow 2n+1 = 9 \Rightarrow \boxed{x=4}$

$n=4 \Rightarrow 2x+1 = 16$ (F)

4 p.

b) i) $\frac{1}{n+1}\cdot\left(1 + \frac{n-2}{n\cdot n+2}\right) = \frac{1}{n+1}\cdot\frac{n\cdot n+2+n-2}{n\cdot n+2} = \dots = \frac{n}{n\cdot n+2}.$

ii) $\frac{247}{420} = \frac{1}{2}\left(1+\frac{37}{210}\right) = \frac{1}{2}\left[1+\frac{1}{6}\left(1+\frac{2}{35}\right)\right] = \frac{1}{2}\left\{1+\frac{1}{6}\left[1+\frac{1}{18}\left(1+\frac{1}{35}\right)\right]\right\} =$

$= \frac{1}{2} + \frac{1}{2\cdot 6} + \frac{1}{2\cdot 6\cdot 18} + \frac{1}{2\cdot 6\cdot 18\cdot 35}$ și altă desc. $\frac{1}{4} + \frac{1}{6} + \frac{1}{10} + \frac{1}{14}$

3 p.

3a) Fie $a < b \Leftrightarrow b - a = 1 \Rightarrow b = a+1$

$d \ln |c-d| = 1 \Rightarrow d - c = 1 \Rightarrow d = c+1$ dar $a^2+b^2+c^2 = d^2$ cube $\Rightarrow$

$a^2 + (a+1)^2 + c^2 = (c+1)^2 \Leftrightarrow \dots \Leftrightarrow a(a+1) = c$

$V = a\cdot b\cdot c = a\cdot(a+1)\cdot a(a+1) = [a(a+1)]^2$

3 p.

$|V - 2014| = \min$.

Fie $V = (6\cdot 7)^2 = 42^2 = 1764 \Rightarrow V = 42^2 = 1764$

sau $V = (7\cdot 8)^2 = 56^2 = 3136$

2 p.

b) $V = a\cdot b\cdot c = 6\cdot 7\cdot c = 42^2 \Rightarrow c = 42$.

$\Rightarrow d = c+1 = 43$ ✓

sau $d = \sqrt{a^2+b^2+c^2} = \dots = 43$.

2 p.